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Developing a System of Nonlinear Delay Differential Equations Solved by Using Picard Method

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الخلاصه:-

الغرض من هذا البحث هو عرض الحلول التقريبية لمنظومة المعادلات التفاضلية التباطؤية اللاخطية باستخدام طريقة بيكارد. خاصية هذه الطريقة ادت الى نتائج جيدة تم توضيحها بالأمثلة.

Abstract :-

The purpose of this paper is to survey the approximated solution for a system of non-linear delay differential equations by using Picard method. Moreover, this method property leads to good results, which are demonstrated, with some examples.

Keywords:-Nonlinear delay differential equation, Approximated Solution, Picard Method.

Introduction:-

Delay differential equations (DDEs) are differential equations having delay argument. They arise in many realistic models of problems in science , engineering and medicine, only in the last few years has much effort in behavior of solution of delay differential equations[1] ,i.e. equations in which the highest order derivative of unknown function appear both with and without delay which called Neutral delay differential equation and equations whith the highest order derivative of unknown function appear without delay which called Retarded differential equation.[2][3].In this paper we study the behavior of the solution of nonlinear delay differential equations of the form

$$f(t, y(t), y(t-\tau_1),..., y(t-\tau_k), y'(t), y'(t-\tau_1),..., y'(t-\tau_k),$$
$$y^n(t), y^n(t-\tau_1)..., y^n(t-\tau_k)) = 0$$

… (1)

Where f is given function and $(\tau_{1,} ...., \tau_k)$ are called delay arguments [3].

2-Picard Method:-

A Picard method is an approximation method which can be applied to the solution of linear or non linear differential equations [4] [5] [6].

The solution is found as infinite series which converges rapidly to accurate solution. Here in this research we explain the Picard Method for finding approximate solution for nonlinear DDEs.

3- The Solution Of A Single First Order Nonlinear Delay Differential Equation

Consider the following first order nonlinear delay differential equation

$$y'(t) = f(t, y(t), y(t-\tau), y'(t-\tau)), t \in [t_{0,\infty}) \quad .... (2)$$

With initial function

$$y(t) = \phi(t) \quad \text{For} \quad t_0 - \tau \le t \le t_0$$

Using Picard Method

Equ. (2) may be solved by Picard method if we use the initial function.

$$y'(t) = f(t, y(t), \phi(t-\tau), \phi'(t-\tau)) \quad … (3)$$

With initial condition

$$y(t_0) = \phi(t_0) \quad ... (4)$$

Integrating the terms of equ.(3) from t_0 to t and taking into account that $y(t_0) = \phi(t_0) = \phi_0(t)$ we get

$$y(t) = \phi_0(t) + \int_{t_0}^{t} f(s, y(s), \phi(s-\tau), \phi'(s-\tau))ds \qquad \dots. (5)$$

We will take $\phi_0(t)$ for the zeroth approximation of the solution. Substituting $\phi_0(t)$ into the integrand on the right of equ. (5) in place ϕ, we get

$$y_1(t) = \phi_0(t) + \int_{t_0}^{t} f(s, \phi_0(s), \phi_0(s-\tau), \phi_0'(s-\tau))ds$$

This is the first approximation of the differential equation (3) that satisfies the initial conditions (4).
Substituting the first approximation $\phi_1(t) = y_1(t)$ into the integrand in equ.(5), we get

$$y_2(t) = \phi_0(t) + \int_{t_0}^{t} f(s, \phi_1(s), \phi_1(s-\tau), \phi_1'(s-\tau))ds$$

This is the second approximation. By continuing this process we obtain the nth approximation y_n (t) as:

$$y_n(t) = \phi_0(t) + \int_{t_0}^{t} f(s, \phi_{n-1}(s), \phi_{n-1}(s-\tau), \phi_{n-1}'(s-\tau))dt$$

Finally, $y_n(t) \to y(t)$ as $n \to \infty$.

The following algorithm summarizes the step for finding approximated solution for different type nonlinear delay differential equation.

Algorithm (SAP1)

Step 1:

(a) Input N "Define the number of iteration"

(b) Input f "Define the differential equation"

(c) Input ϕ_0 "Define the initial conditions"

(d) Input t_0 "Define the initial interval"

Step 2:

$$y_{j+1} = \phi_0 + \mathrm{int}(f(s, \phi_j(s), \phi_j(s-\tau), \phi_j'(s-\tau)), 't', t_0, t)$$

For each $j = 0,1,\ldots, N$

<u>4- Soution of a System of First Order Nonlinear Delay Differential Equation:</u>

Consider a system of first order nonlinear delay differential equation such as:

$$\frac{dy_i}{dt} = f_i(t, y_1(t), \ldots, y_n(t), y_1(t-\tau_1), \ldots, y_n(t-\tau_n), y_1'(t-\tau_1), \ldots, y_n'(t-\tau_n))$$

$i = 0,1,\ldots, N$

… (6)

With initial function

$$y_i(t) = \phi_i(t) \quad \text{for} \quad \begin{matrix} t_0 - \tau_i \le t \le t_0 \\ i = 0,1,\ldots, N \end{matrix}$$

<u>Using Picard Method</u>

Equation (6) may be solved by Picard method if

$$\frac{dy_i}{dt} = f_i(t, y_1(t), \ldots, y_n(t), \phi_1(t-\tau_1), \ldots, \phi_n(t-\tau_n), \phi_1'(t-\tau_1), \ldots, \phi_n'(t-\tau_n))$$

… (7)

with initial condition

$$y_i(t_0) = \phi_i(t_0) \quad \text{for} \quad i = 0,1,....,N$$

Integrating the terms of equ. (7) From t_io to t $(i = 0,1,....,N)$ and taking into account that $y_i(t_0) = \phi_i(t_0) = \phi_{i0}$, we get

$$y_i(t) = \phi_{i0}(t) + \int_{t_{i0}}^{t} f_i(s, y_1(s),...,y_n(s), \phi_1(s-\tau_1),...,$$
$$\phi_n(s-\tau_n), \phi_1'(s-\tau_1),...,\phi_n'(s-\tau_n))ds \qquad \text{… (8)}$$

We will take ϕ_{i0} $(i = 0,1,....,N)$ for the zeroth approximation

Substituting ϕ_{i0} $(i = 0,1,....,N)$ into the integrand on the right of equ.(2.8), we get

$$y_{i,1}(t) = \phi_{i0}(t) + \int_{t_{i0}}^{t} f_i(s, y_{10}(t),...,y_{n0}(t), \phi_{10}(t-\tau_1),...,$$
$$\phi_{n0}(t-\tau_n), \phi_{10}'(t-\tau_1),...,\phi_{n0}'(t-\tau_n))dt$$

for each $i = 0,1,....,N$

This is the first approximation of the differential equation (7) that satisfies the initial conditions (7).

Substituting the first approximation $\phi_{i1}(t) = y_{i1}(t)$ $(i = 1,2,....,N)$ into the integrand in equ.(8), we get

$$y_{i,2}(t) = \phi_{i0}(t) + \int_{t_{i0}}^{t} f_i(s, y_{11}(t),..., y_{n1}(t), \phi_{11}(t-\tau_1),...,$$
$$\phi_{n1}(t-\tau_n), \phi_{11}'(t-\tau_1),..., \phi_{n1}'(t-\tau_n))dt$$

This is the second approximation. By continuing this process we obtain the mth approximation $y_{i,m}(t)$ as:

$$y_{i,m}(t)=\phi_{i0}(t)+\int_{t_{i0}}^{t} f_i(s,y_{1,m-1}(t),\dots,y_{n,m-1}(t),\phi_{1,m-1}(t-\tau_1),\dots,$$
$$\phi_{n,m-1}(t-\tau_n),\phi'_{1,m-1}(t-\tau_1),\dots,\phi'_{n,m-1}(t-\tau_n))dt$$

The following algorithm summarized the step for finding approximated solution for system nonlinear delay differential equations.

<u>Algorithm (SAPS)</u>

Step 1:

a. Input M "Define the number of iteration"
b. Input f "Define the differential equation "
c. Input ϕ_{i0} "Define the initial conditions" for $i=1,2,\dots.,N$
d. Input t_{i0} "Define the initial intervals" for $i=1,2,\dots.,N$

Step 2:

$$y_{i,j}=\phi_{i0}(t)+\text{int}(f_i(s,y_{1,j-1}(s),\dots,y_{n,j-1}(s),\phi_{1,j-1}(s-\tau_1),\dots,$$
$$\phi_{n,j-1}(s-\tau_n),\phi'_{1,j-1}(s-\tau_1),\dots,\phi'_{n,j-1}(s-\tau_n)),'t',t_{i0},t)$$

for each $j=1,2,\dots.,M,\quad i=1,2,\dots.,N$

<u>5- Test Examples</u>

<u>Example (1):-</u>

Consider the following nonlinear Retarded of DDE

$$y'(t)=\frac{1}{t}\exp(y(y(t)-\ln 2+1)) \qquad t\ge 1$$

where $y(t)=0 \qquad t\le 0$

The analytic Solution: $y(t)=\ln t \qquad 1\le t\le 2$.

The Picard method is used this problem represented by the program SAP1. The results and the least square error (L.S.E.) for this program are listed in table (1).

Table (1) the solution of example (1)

t	exact	SAP1
1	0	0
1.1	0.09531	0.09531
1.2	0.18232	0.18232
1.3	0.26236	0.26236
1.4	0.33647	0.33647
1.5	0.40547	0.40547
1.6	0.47000	0.47000
1.7	0.53063	0.53063
1.8	0.58779	0.58779
1.9	0.64185	0.64185
2	0.69315	0.69315
L.S.E.		0.00000

Example (2):-

Consider the following nonlinear Neutral of DDE

$$y'(t) = \cos t\,(1 + y(ty(t)^2) + y(t)y'(ty(t)^2) - \sin(t + t\sin^2 t) \qquad t \geq 0$$

where $y(t) = 0 \qquad t \leq 0$

The analytical solution

$$y(t) = \sin t \qquad 0 \leq t \leq 1$$

The Picard method is used this problem represented by the program SAP1. The results and the least square error (L.S.E.) for this program are listed in table (2).

Table (2) The solution of example (2)

t	exact	SAP1

0	0	0
0.1	0.09983	0.09983
0.2	0.19867	0.19867
0.3	0.29552	0.29552
0.4	0.38942	0.38942
0.5	0.47943	0.47943
0.6	0.56464	0.56464
0.7	0.64422	0.64422
0.8	0.71736	0.71736
0.9	0.78333	0.78333
1	0.84147	0.84147
L.S.E.		0.00000

Example (3)

Consider the following nonlinear system of DDE

$$y_1'(t) = y_2(t-1)y_1(t-1) \qquad t \geq 0$$

$$y_2'(t) = y_2(t-1) \qquad t \geq 0$$

where

$$y_1(t) = \exp(t+1) \qquad t \leq 0$$

$$y_2(t) = \exp(t+1) \qquad t \leq 0$$

The analytical solution

$$y_1(t) = \frac{1}{2}\exp(2t) - \frac{1}{2} + \exp(1)$$

$$0 \leq t \leq 1$$

$$y_2(t) = \exp(t) - \exp(1) + 1$$

The Picard method is used this problem represented by the program SAPS. The results and the least square error (L.S.E.) for this program are listed in table (3).

Table (3) The solution of example (3)

T	Exact1	SAPS1	Exact2	SAPS2
0	2.718	2.718	2.718	2.718
0.1	2.829	2.829	2.823	2.823
0.2	2.964	2.964	2.940	2.940
0.3	3.129	3.129	3.068	3.068
0.4	3.331	3.331	3.210	3.210
0.5	3.577	3.577	3.367	3.367
0.6	3.878	3.878	3.540	3.540
0.7	4.246	4.246	3.732	3.732
0.8	4.695	4.695	3.944	3.944
0.9	5.243	5.243	4.178	4.178
1	5.913	5.913	4.437	4.437
L.S.E		0.000	L.S.E	0.000

6-Conclusions:-

The method, which was described, provides convenient and efficient way for solving different type of first order nonlinear delay differential equation as well as a system of nonlinear delay differential equation analytically.

7-Refference:-

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